Sheet 2

Handed out on 26. 10. 17 for the Tutorial on 09. 11. 17

Problem 4: Lambert-Beer's law and absorption cross sections (3P)

In a YAG crystal ($Y_3Al_5O_{12}$) the Y^{3+} ions can be exchanged by laser-active ions to realize a laser medium. YAG has a cubic crystal structure, a mass density of $4.55\ \frac{\mathrm{g}}{\mathrm{cm}^3}$ and $N_A = 6.023 \times 10^{23}\ \mathrm{mol}^{-1}$ YAG formula units correspond to a molar mass of $M = 593.6\ \frac{\mathrm{g}}{\mathrm{mol}}$.

(a) Show that for a homogeneous doping concentration of 1% of Er^{3+} ions, i.e. a replacement of one Y^{3+} ion by one Er^{3+} ion on 99 Y^{3+} ions, the number density of dopant ions is given by $N \approx 1.38 \times 10^{26}\ \mathrm{m}^{-3}$. (2P)

(b) For a $L = 10\ \mathrm{mm}$-long 0.5% Er^{3+}:YAG crystal on its maximum absorption line at $1532\ \mathrm{nm}$ a transmission of $T = 20.5\%$ is measured. Calculate the absorption cross section (1P).

Problem 5: Emission cross section and Füchtbauer-Ladenburg relation (4P)

As long as only homogeneous broadening effects are present, the spectral shape of a transition line is given by a Lorentzian line form function

$$g(\nu) = \frac{2}{\pi} \frac{\Delta\nu}{4(\nu - \nu_0)^2 + \Delta\nu^2} \ . \qquad (1)$$

(a) Starting from the line form function in frequency space $g(\nu)d\nu$, show that the line form function in wavelength space is again a Lorentzian function

$$g(\lambda) = \frac{2}{\pi} \frac{\Delta\lambda}{4(\lambda - \lambda_0)^2 + \Delta\lambda^2} \qquad (2)$$

when we assume $\Delta\lambda \ll \lambda_0$. (1P)

These functions are normalized according to

$$\int_\nu g(\nu)d\nu = 1 \quad \text{and} \quad \int_\lambda g(\lambda)d\lambda = 1 \ . \qquad (3)$$

As long as the line width $\Delta\lambda$ is small compared to the center wavelength λ_0, one will also find a Lorentzian shape for the corresponding emission cross section, i.e.

$$\sigma_e(\lambda) = \sigma_e(\lambda_0) \frac{\Delta\lambda^2}{4(\lambda - \lambda_0)^2 + \Delta\lambda^2} \qquad (4)$$

(b) Derive the following expression for the product of the peak emission cross section and the lifetime (2P)

$$\sigma_e(\lambda_0)\tau = \frac{\lambda_0^4}{4\pi^2 n^2 c\Delta\lambda} \ . \tag{5}$$

(c) Using the relation between the homogeneous line width and the upper state lifetime, show that the peak emission cross section of a transition which shows only its natural linewidth, e.g. for a free atom in vacuum, can be approximated by (1P)

$$\sigma_e(\lambda_0) = \frac{\lambda_0^2}{2\pi} \ . \tag{6}$$

Problem 6: McCumber relation and quasi-three-level lasers (4P)

The Yb^{3+} ion is a typical quasi-three-level laser ion. It is mostly used in combination with the YAG crystal, especially in modern disc lasers. Due to its simple quantum-mechanical structure, Yb^{3+} only shows two manifolds, the ground-state manifold ${}^4F_{7/2}$ and the excited state manifold ${}^4F_{5/2}$. In YAG, the ${}^4F_{5/2}$ fluorescence lifetime is $951\ \mu$s and the different Stark levels of the manifolds have been determined at $T = 300$ K to

$$\begin{array}{rcl} & {}^4F_{5/2} & \\ E_{2,3} & = & 10679\ \mathrm{cm}^{-1} \\ E_{2,2} & = & 10624\ \mathrm{cm}^{-1} \\ E_{2,1} & = & 10327\ \mathrm{cm}^{-1} \\ & {}^4F_{7/2} & \\ E_{1,4} & = & 785\ \mathrm{cm}^{-1} \\ E_{1,3} & = & 612\ \mathrm{cm}^{-1} \\ E_{1,2} & = & 565\ \mathrm{cm}^{-1} \\ E_{1,1} & = & 0\ \mathrm{cm}^{-1} \end{array}$$

with a degeneracy of 2. The energy is given in cm^{-1}, which can be transformed into SI energy units by multiplying it with a factor hc. The main pump wavelength is $\lambda_p = 941\ \mathrm{nm}$, with a pump absorption cross section of $\sigma_a(\lambda_p) = 7.6 \times 10^{-21}\ \mathrm{cm}^2$ at $T = 300\ \mathrm{K}$. The mostly used laser wavelength is $\lambda_s = 1029\ \mathrm{nm}$, with an emission cross section of $\sigma_e(\lambda_s) = 2.31 \times 10^{-20}\ \mathrm{cm}^2$ at $T = 300\ \mathrm{K}$.

(a) Find the Stark levels of the pump and the laser transition and the Boltzmann population factors of the starting levels. (2P)

(b) Calculate the chemical potential wavelength between the two manifolds. (1P)

(c) Calculate the laser reabsorption cross section and the pump backemission cross section. (1P)